4.1 Trig Ratios of Obtuse Angles

Trig Ratios of Obtuse Angles [4.1]

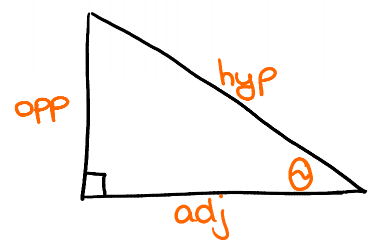
Warm Up:

- Write down the Primary Trig Ratios SOH CAH TOA
- Draw the type of triangle that they apply to
- Identify the sides of the triangle referred to in the ratios

$\sin\theta = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp}$

$\tan\theta = \frac{opp}{adj}$



⇨ What does "sin 80° = 0.9848…." mean?

↳ $\sin 80° = \frac{opp}{hyp}$ → the ratio between the lengths of the opposite side and the hypotenuse of a right triangle.

In small groups, complete the table provided on the back of this page.

Here are some hints to guide you:

- Do you see similarities when you look at the sine ratios or just the cosine ratios?
- Do you see any patterns when you look at the ratios for the supplementary angles?

Before you go, here's what you need to know:

- The sine ratios for supplementary angles are equal.
- The cosine and tangent ratios for supplementary angles are opposites.

$\sin\theta = \sin(180° - \theta)$

$\cos\theta = -\cos(180° - \theta)$

$\tan\theta = -\tan(180° - \theta)$

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
100°	0.9848	-0.1736	-5.6713
110°	0.9397	-0.3420	-2.7475
120°	0.8660	-0.5000	-1.7321
130°	0.7660	-0.6428	-1.1918
140°	0.6428	-0.7660	-0.8391
150°	0.5000	-0.8660	-0.5774
160°	0.3420	-0.9397	-0.3640
170°	0.1736	-0.9848	-0.1763
180°	0	-1	0

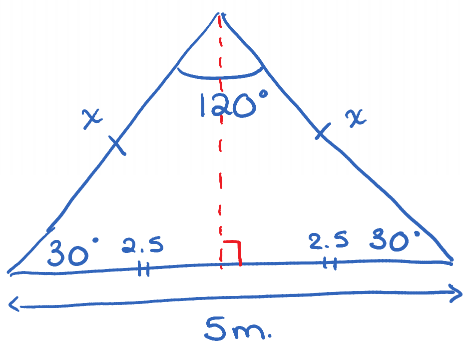
$(180° - \theta)$	$\sin(180° - \theta)$	$\cos(180° - \theta)$	$\tan(180° - \theta)$
80°	0.9848	0.1736	5.6713
70°	0.9397	0.3420	2.7475
60°	0.8660	0.5000	1.7321
50°	0.7660	0.6428	1.1918
40°	0.6428	0.7660	0.8391
30°	0.5000	0.8660	0.5774
20°	0.3420	0.9397	0.3640
10°	0.1736	0.9848	0.1763
0°	0	1	0

4.2 (1) Sine & Cosine Law for Obtuse Angles

Sine & Cosine Law for Obtuse Angles – Part 1 [4.2]

<u>Explore:</u> An isosceles obtuse triangle has one angle that measures 120° and one side length that is 5m. What could the other side lengths be?

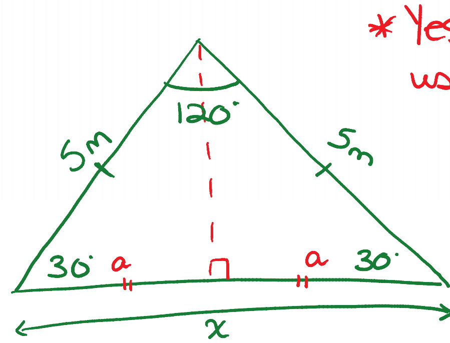
(Think about: What do you know about Isos Δs? How many Δs could you draw? What happens if you look at the height of the Δ and then the relationship with the 2 smaller Δs?)



$\cos 30° = \frac{adj}{hyp}$

$\cos 30° = \frac{2.5}{x}$

$x = \frac{2.5}{\cos 30°} \rightarrow \boxed{x = 2.8867\text{ m}}$



* Yes you can use cosine law too!

$\cos 30° = \frac{adj}{hyp}$

$\cos 30° = \frac{a}{5}$

$a = 5(\cos 30°)$

$a = 4.3301\ldots$

$x = 2a \rightarrow \boxed{x = 8.660\text{ m}}$

<u>A Few Notes ...</u>

- Be careful when using the sine law to determine the measure of an angle. The inverse sine of a ratio always gives an acute angle, but the supplementary angle has the same ratio. You must decide whether the acute angle, θ, or the obtuse angle, $180° - \theta$, is the correct angle for your triangle.
- Because the cosine ratios for an angle and its supplement are not equal (they are opposites), the measures of the angles determined using the cosine law are always correct.

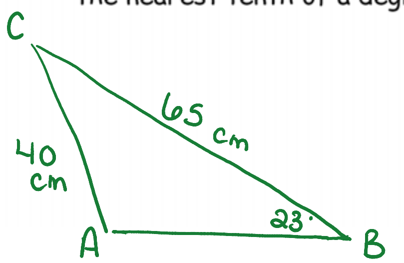
<u>Sine Law</u>

Obtuse ∠

$180° - \theta$

Example 1:

In an obtuse triangle, ∠ B measures 23.0° and its opposite side, *b*, has a length of 40.0 cm. Side *a* is the longest side of the triangle, with a length of 65.0 cm. Determine the measure of ∠ A to the nearest tenth of a degree.



$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{40}{\sin 23^\circ} = \frac{65}{\sin A}$$

$$40(\sin A) = 65(\sin 23^\circ)$$

$$\sin A = \frac{65(\sin 23^\circ)}{40}$$

$$\angle A = \sin^{-1}\left(\frac{65(\sin 23^\circ)}{40}\right)$$

$$\angle A = 39.4^\circ$$

↳ Is this obtuse? No! So

$180^\circ - 39.4^\circ$

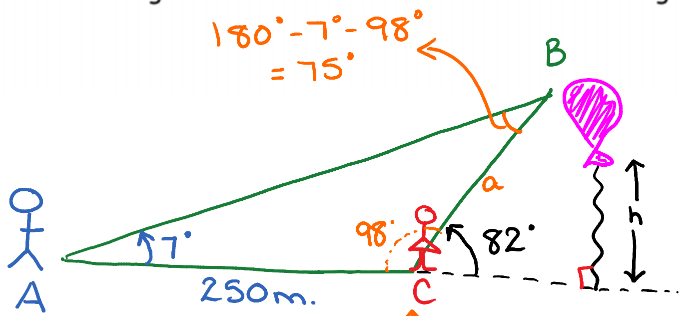
$= 140.6^\circ$

Is this obtuse? Yes!

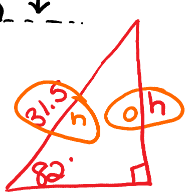
$$\boxed{\angle A = 140.6^\circ}$$

Example 2:

Colleen and Juan observed a tethered balloon advertising the opening of a new fitness centre. They were 250m apart, joined by a line that passed directly below the balloon, and were on the same side of the balloon. Juan observed the balloon at an angle of elevation of 7° while Colleen observed the balloon at an angle of elevation of 82°. Determine the height of the balloon to the nearest metre.



if $98^\circ / 82^\circ$



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin 7^\circ} = \frac{250}{\sin 75^\circ}$$

$$a = \frac{250(\sin 7^\circ)}{\sin 75^\circ}$$

$$\boxed{a = 31.5\text{ m}}$$

$$\sin 82^\circ = \frac{opp}{hyp}$$

$$\sin 82^\circ = \frac{h}{31.5}$$

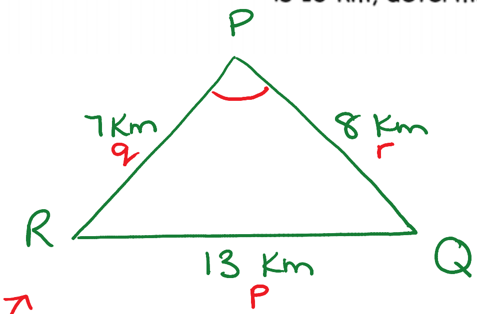
$$h = 31.5(\sin 82^\circ)$$

$$\boxed{h = 31\text{m}}$$

4.2 (2) Sine & Cosine Law for Obtuse

Sine & Cosine Law for Obtuse Angles – Part 2 [4.2]

Example 1: Two ships set sail from, P, heading in different directions. The first ship sails 7 km to R and the second ship sails 8 km to Q. If the distance between R and Q is 13 km, determine the angle between the directions of the two ships.



All 3 sides as clues

$$p^2 = q^2 + r^2 - 2qr\cos P$$
$$13^2 = 7^2 + 8^2 - 2(7)(8)\cos P$$
$$169 = 49 + 64 - 112\cos P$$
$$169 - 113 = 113 - 113 - 112\cos P$$

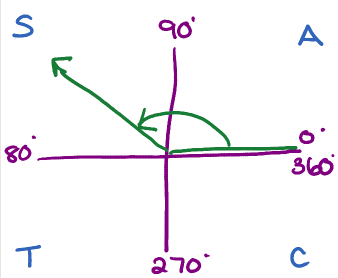
* Bedmas*

$$\frac{56}{-112} = \frac{-112\cos P}{-112}$$
$$\cos P = -0.5$$
$$P = \cos^{-1}(-0.5)$$

* What does this ratio tell us?

$$\boxed{P = 120°}$$

Extra Info



A = all ratios are positive

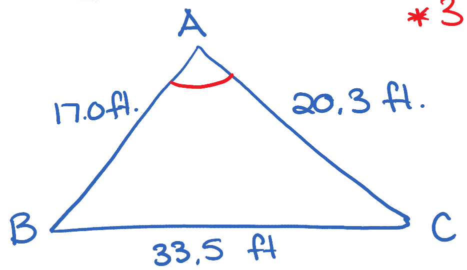
S = only sine ratios are positive

T = only tan ratios are positive

C = only cosine ratios are positive

∴ a neg Cos Ratio → we have an obtuse angle! ($\theta > 90°$)

Example 2: The roof of a house consists of two slanted sections, as shown. A roofing cap is being made to fit the crown of the roof, where the two slanted sections meet. Determine the measure of the angle needed for the roofing cap, to the nearest tenth of a degree.



*3 sides as clues... Use Cosine Law!

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$33.5^2 = \underbrace{20.3^2 + 17^2} - \underbrace{2(20.3)(17)} \cos A$$

$$1122.25 = 701.09 - 690.2 \cos A$$

$$\frac{421.16}{-690.2} = \frac{-\cancel{690.2} \cos A}{-\cancel{690.2}}$$

$$\cos A = \left(\frac{421.16}{-690.2}\right)$$

$$A = \cos^{-1}\left(\frac{421.16}{-690.2}\right)$$

*What does the neg. tell us? (Obtuse ∠)

$$\boxed{A = 127.6^\circ}$$

4.3 (1) The Ambiguous Case

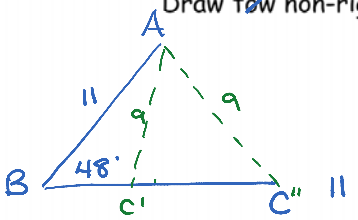
Sine Law - Part 1 [4.3]

When given two sides and an angle <u>other than the contained angle</u>,

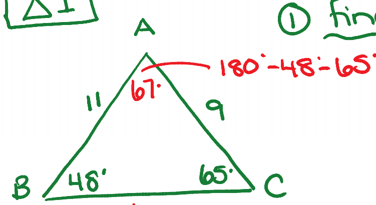
there are two possible triangles.

Draw ~~tow~~ 2 non-right triangles that fit the following criteria, and solve each triangle.

AB = 11 ∠ B = 48° AC = 9



$\boxed{\Delta 1}$



① Find ∠C"

$\frac{\sin C}{c} = \frac{\sin B}{b}$

$\frac{\sin C}{11} = \frac{\sin 48°}{9}$

$\sin C = \frac{11(\sin 48°)}{9}$

$\angle C'' = \sin^{-1}\left(\frac{11(\sin 48°)}{9}\right)$

$\angle C'' = 65°$ → acute

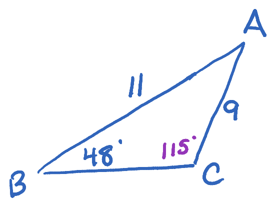
$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 67°}{a} = \frac{\sin 48°}{9}$

$a = \frac{9(\sin 67°)}{\sin 48°}$

$a = 11$

$\boxed{\Delta 2}$



For Obtuse ∠C take

$180° - 65° = 115°$ → Isos. Δ

$(\sin 65° = \sin 115°)$

$\angle A = 180° - 48° - 115°$

$\angle A = 17°$

$\frac{\sin 17°}{a} = \frac{\sin 48°}{9}$

$a = \frac{9(\sin 17°)}{\sin 48°}$

$a = 3.5$

If Δ ABC is known to exist, and the measure of ∠ B, the length of side b and side c are given…

Exactly two <u>different</u> triangles satisfy these conditions provided **c(Sin B) < b < c**.

Otherwise there is exactly one triangle.

$11(\sin 48) < 9 < 11$

$8.17 < 9 < 11$ ✓

(side-side-angle)

Given each *SSA* situation for $\triangle ABC$, determine how many triangles are possible.

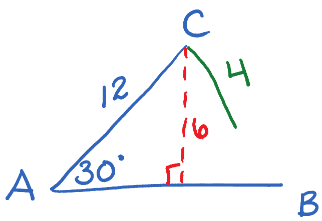
a) $\angle A = 30°$, $a = 4$ m, and $b = 12$ m

b) $\angle A = 30°$, $a = 6$ m, and $b = 12$ m

c) $\angle A = 30°$, $a = 8$ m, and $b = 12$ m

d) $\angle A = 30°$, $a = 15$ m, and $b = 12$ m

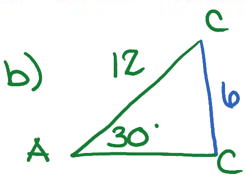
a)



$a = 4 \quad h = 6$

$\boxed{a < h \text{ No Triangle}}$

b)



$a = 6$

$h = 6$

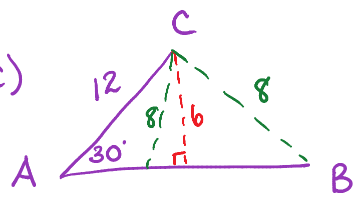
$\boxed{a = h \text{ one right } \triangle}$

$h \Rightarrow \sin 30 = \frac{x}{12}$

$x = 12 \sin 30°$

$x = 6$

c)



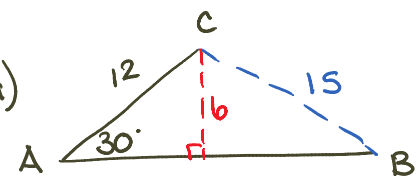
$a = 8$

$h = 6$

$b = 12$

$\boxed{\begin{array}{c} a > h \\ a < b \\ \text{Two Triangles} \end{array}}$

d)



$\boxed{\begin{array}{c} a > h \\ a > b \\ \text{One Triangle} \end{array}}$

In Summary

Key Idea

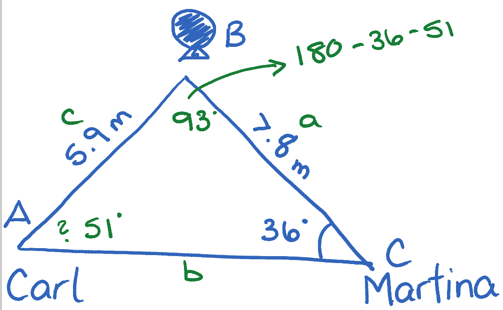
- The ambiguous case of the sine law may occur when you are given two side lengths and the measure of an angle that is opposite one of these sides. Depending on the measure of the given angle and the lengths of the given sides, you may need to construct and solve zero, one, or two triangles.

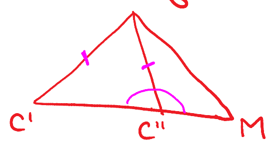
4.3 (2) The Ambiguous Case

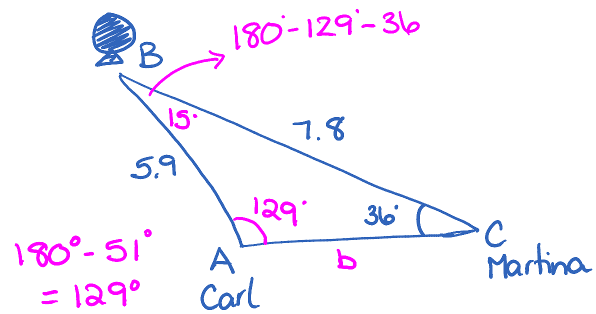
Sine Law: The Ambiguous Case – Part 2 [4.3]

Martina & Carl are part of a team that is studying weather patterns. The team is about to launch a weather balloon to collect data. Martina's rope is 7.8 m long and makes an angle of 36.0° with the ground. Carl's rope is 5.9 m long. Assuming that Martina & Carl form a triangle in a vertical plane with the weather balloon, what is the distance between Martina & Carl, to the nearest tenth of a metre?

2 possibilities: Opposite sides of the balloon (acute angles)
or Same side of the balloon (obtuse angle)







$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin A}{7.8} = \frac{\sin 36^\circ}{5.9}$$

$$\sin A = \frac{7.8(\sin 36^\circ)}{5.9}$$

$$\angle A = \sin^{-1}(0.7771)$$

$$\boxed{\angle A = 51^\circ}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 93^\circ} = \frac{5.9}{\sin 36^\circ}$$

$$b = \frac{5.9(\sin 93^\circ)}{\sin 36^\circ}$$

$$b = 10.02$$

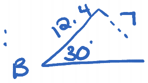
$$\boxed{b = 10.0\text{ m.}}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 15^\circ} = \frac{5.9}{\sin 36^\circ}$$

$$b = \frac{5.9(\sin 15^\circ)}{\sin 36^\circ}$$

$$\boxed{b = 2.6\text{ m}}$$

ex: 

$c(\sin B) < b < c$

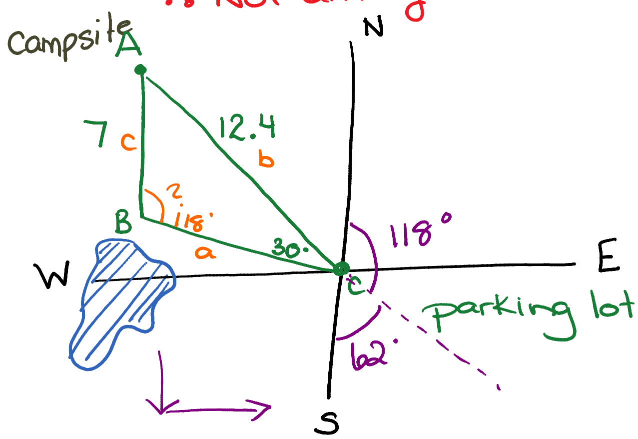
$12.4(\sin 30^\circ) < 7 < 12.4$

$6.2 < 7 < 12.4$ ✓

height

Leanne & Kerry are hiking in the mountains. They left Leanne's car in the parking lot and walked northwest for 12.4 km to a campsite. Then they turned due south and walked another 7.0 km to a glacier lake. The weather was taking a turn for the worse, so they decided to plot a course directly back to the parking lot. Kerry remembered, from the map in the parking lot, that the angle between the path to the campsite and the path to the glacier lake measures about 30°. What compass direction should they follow to return directly to the parking lot? * If we use our 'test' we learn their could be 2 possible triangles but Word Problem tells us only 1 possible Δ *

∴ Not ambiguous - only 1 Δ



$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin B}{12.4} = \frac{\sin 30^\circ}{7}$$

$$\sin B = \frac{12.4(\sin 30^\circ)}{7}$$

$$\angle B = \sin^{-1}\left(\frac{12.4(\sin 30^\circ)}{7}\right)$$

$\angle B = 62^\circ \rightarrow$ acute but we need obtuse

$$\boxed{180^\circ - 62^\circ = 118^\circ}$$

For Compass Direction

S62°E

possible answer

Before you go

Explain what is meant by the ambiguous case of the sine law. Describe situations in which a sine law problem may have no solution, one solution or two solutions.

The ambiguous case of the Sine Law occurs when the given clues allows two different triangles to be drawn generating two different solutions to the problem.

We have 2 different Triangles when

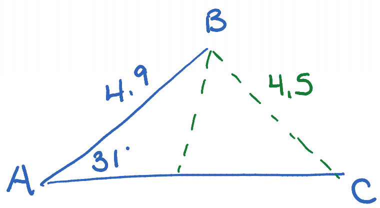
$c(\sin B) < b < c$

4.4 Solving Problems using Obtuse Triangles

Solving Problems using Obtuse Triangles [4.4]

Quick Review:

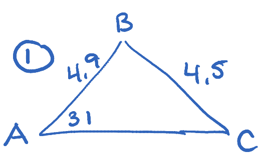
Find all possible measures of $\angle C$ in the following triangles:



a) Δ ABC where $\angle A = 31°$, a = 4.5 cm and c = 4.9 cm

$c(\sin A) < a < c$

$4.9(\sin 31°) < 4.5 < 4.9$ ✓ > 2 possible Δs / solutions

① 

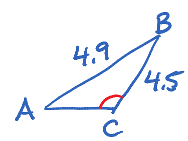
$\rightarrow \dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

$\dfrac{\sin C}{4.9} = \dfrac{\sin 31°}{4.5}$

$\sin C = \dfrac{4.9(\sin 31°)}{4.5}$

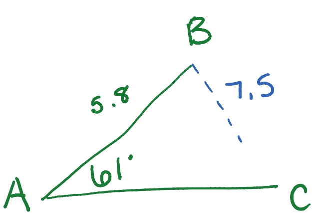
$C = \sin^{-1}\left(\dfrac{4.9(\sin 31°)}{4.5}\right)$

$\boxed{C = 34°}$



OR $180° - 34°$

$\boxed{\angle C = 146°}$



b) Δ ABC where $\angle A = 61°$, a = 7.5 cm, and c = 5.8 cm

$c(\sin A) < a < c$?

$5.8(\sin 61°) < 7.5 < 5.8$ X > Nope! 1 Δ, 1 solution

$\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

$\dfrac{\sin C}{5.8} = \dfrac{\sin 61°}{7.5}$

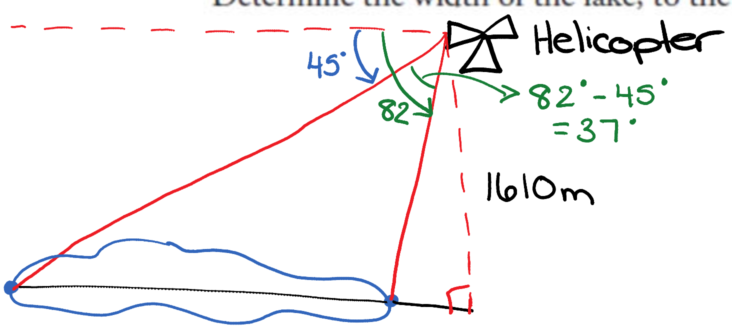
$\sin C = \dfrac{5.8(\sin 61°)}{7.5}$

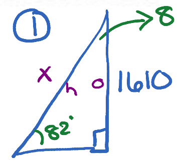
$C = \sin^{-1}\left(\dfrac{5.8(\sin 61°)}{7.5}\right)$

$\boxed{C = 43°}$

A surveyor in a helicopter would like to know the width of Garibaldi Lake in British Columbia. When the helicopter is hovering at 1610 m above the forest, the surveyor observes that the angles of depression to two points on opposite shores of the lake measure 45° and 82°. The helicopter and the two points are in the same vertical plane.

Determine the width of the lake, to the nearest metre.



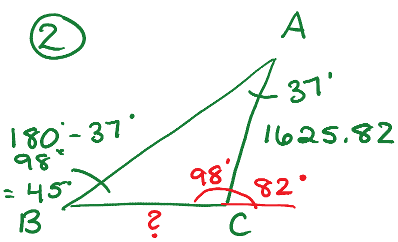


$\sin 82° = \frac{opp}{hyp}$

$\sin 82° = \frac{1610}{x}$

$x = \frac{1610}{\sin 82°}$

$\boxed{x = 1625.82}$



$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{a}{\sin 37°} = \frac{1625.82}{\sin 45°}$

$a = \frac{1625.82(\sin 37°)}{\sin 45°}$

$\boxed{a = 1384\ m}$

Practice Assignments: p. 194 #3, 4, 6, 13 Bonus #11